Technology of perfect substitutes

Consider the following production function: $f(K, L) = aK + bL$ where $a, b > 0$.

1. Obtain the marginal factor productivities. Interpret.
2. Verify if the law of diminishing marginal returns holds. Interpret.
3. Obtain the technical rate of substitution and interpret.
4. Determine the returns to scale for this technology.

Solutions

1. To obtain the marginal productivities, we take the derivative of the production function with respect to each input: $f'_K = a$ and $f'_L = b$. The marginal productivities are positive, so an increase in either the capital or labor input leads to an increase in production.

2. The second derivatives are: $f''_{KK} = 0$ and $f'_{LL} = 0$. This tells us that an increase in either K or L always generates the same increase in production; it neither decreases nor increases through more or fewer input increases. The law of diminishing marginal returns does not hold since for that to happen, the second derivatives should be negative. This would mean that an increase in the amount of input L or K would increase production by a smaller and smaller amount over time.

3. We find the quotient of the derivatives:
$$\frac{f'_K}{f'_L} = \frac{a}{b}$$
The technical rate of substitution, or the technical substitution rate, tells us how much we need to decrease one input if we increase the other by an infinitesimally small amount so that there is no change in production. Specifically, the calculated TRS tells us that if we increase K by an infinitesimal amount, we need to decrease L by a/b to keep production constant. This is the slope of the isoquant curve for the production function.

4. To calculate the returns to scale, we find the degree of homogeneity.
$$f(hK, hL) = aKh + bhL = h(aK + bL) = hf(K, L)$$
With this result, we can affirm that the production function is homogeneous of degree 1 and therefore exhibits constant returns to scale, i.e., an increase in K and L at the same time leads to a proportional increase in production.